VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

CODE A

JEE Main Home Practice Test - 6 | JEE - 2024

Date: 10/01/2024 **Maximum Marks: 300**

Timing: 10:00 AM to 1:00 PM **Duration : 3.0 Hours**

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300.**
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. You will NOT be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.**
   The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.
6. On completion of the test, the candidate must hand over the Answer Sheet to the **Invigilator** on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them**.

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

Syllabus

Physics: Full Syllabus
Chemistry: Full Syllabus
Mathematics: Full Syllabus

Name of the Candidate (In CAPITALS) : _____

Roll Number : _____

OMR Bar Code Number : _____

Candidate's Signature : _____ **Invigilator's Signature** _____

PART I : PHYSICS 100 MARKS

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE CHOICE is correct.

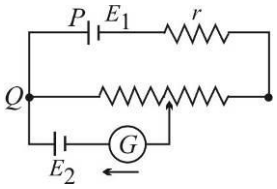
1. A microscope is having objective of focal length 1 *cm* and eye-piece of focal length 6 cm. If tube length is 30 cm and image is formed at the least distance of distinct vision, what is the magnification produced by the microscope? Take least distance of distinct vision = 25 *cm.*

(A) 25 **(B)** 6 **(C)** 125 **(D)** 155

2. An aluminium container of mass 100 gm contains 200 *gm* of ice at – 20°C. Heat is added to the system at the rate of 100 cal/s. The temperature of the system after 4 minutes will be: (specific heat of ice = 0.5Cal/*gm*/°C and $L = 80$ cal/*gm*, specific heat of *Al* = 0.2 Cal/*gm*/°C)

(A) 0°C **(B)** 25.5 °C **(C)** 30.3°C **(D)** 35.0°C

3. In the potentiometer circuit of given figure the galvanometer reveals a current in the direction shown wherever the sliding contact touches the wire. This can not be caused by:



(A) E_1 being too low **(B)** *r* being too high

(C) a break in *PQ* **(D)** E_2 being too low

4. A solid sphere of radius R made of material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, the fractional change in the radius of the sphere $(\delta R / R)$ is:

(A) $\frac{Mg}{2AK}$ **(B)** $\frac{Mg}{3AK}$ **(C)** $\frac{Mg}{4AK}$ **(D)** $\frac{Mg}{2\sqrt{2}K}$

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5. A proton and an α-particle are accelerated through the same potential difference. The ratio of their de Broglie wavelengths is:

(A) $\sqrt{2}$ **(B)** $1/\sqrt{2}$ **(C)** $2\sqrt{2}$ **(D)** 2

6. A liquid takes 30 seconds to cool from 95ºC to 90ºC and 70 seconds to cool from 55 to 55ºC. Find the room temperature and the time it will take to cool from 50ºC to 45ºC

(A) 60 sec **(B)** 84 sec **(C)** 96 sec **(D)** 80 sec

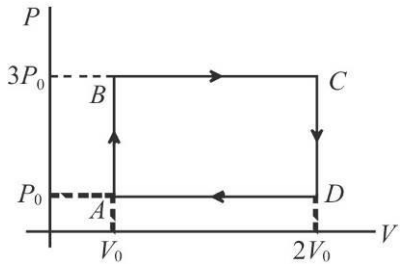
7. In a Young's double-slit experiment, let S_1 and S_2 be the two slits and C be the centre of the screen. If $\angle S_1CS_2 = \theta$ and λ is the wavelength, the fringe width will be: [Assume slits to be close to each other]

(A) $\frac{\lambda}{\theta}$ **(B)** $\theta\lambda$ **(C)** $\frac{2\lambda}{\theta}$ **(D)** $\frac{\lambda}{2\theta}$

8. A block of wood of mass m and specific gravity 0.5 is submerged at a depth of h in a vessel filled with water, which is accelerated upwards with an acceleration $g/2$. The time taken by the block to reach the surface, when released with zero velocity is:

(A) $\sqrt{\frac{2h}{g}}$ **(B)** $\sqrt{\frac{h}{g}}$ **(C)** $2\sqrt{\frac{h}{3g}}$ **(D)** $2\sqrt{\frac{h}{g}}$

9. An ideal monoatomic gas is carried around the cycle *ABCDA* as shown in the figure. The efficiency of the cyclic process is:



(A) $\frac{4}{21}$ **(B)** $\frac{2}{21}$ **(C)** $\frac{4}{31}$ **(D)** $\frac{2}{31}$

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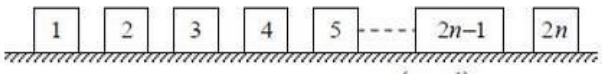
10. $2n$ identical cubical blocks are kept in a straight line on a horizontal smooth surface. The distance between the consecutive blocks is same. The blocks $1,3,5,.....(2n-1)$ are given velocity v to the right whereas blocks $2,4,6,.....2n$ are given velocity to the left. All collisions between blocks are perfectly elastic. The total number of collisions that will take place is:



(A) n **(B)** $n+1$ **(C)** $\frac{n(n+1)}{2}$ **(D)** $n(n+1)$

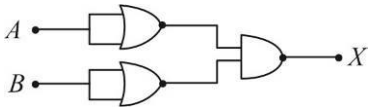
11. A light string is tied to another light string of equal length L and two strings are tied to the fixed supports at their other ends. The strings are set into vibration so that junction point A is a node. The wavelength of the loops formed to the left and to the right of the junction point A is λ_1 and λ_2 respectively. Find the difference of the number of the loops formed to the right and to the left of the point A.

(A) $\frac{L(\lambda_1-\lambda_2)}{2\lambda_1\lambda_2}$ **(B)** $\frac{2L(\lambda_1-\lambda_2)}{\lambda_1\lambda_2}$ **(C)** $\frac{4L(\lambda_1-\lambda_2)}{\lambda_1\lambda_2}$ **(D)** $\frac{4L(\lambda_1+\lambda_2)}{\lambda_1(\lambda_1-\lambda_2)}$

12. A particle is moving on a circular path of radius $\frac{100}{\sqrt{19}}$ m in such a way that magnitude of its velocity varies with time as $v=2t^2+t$, where v is velocity in *m/s* and t is time in *s*. The acceleration of the particle at t = 2 *s* is:

(A) 21 m/s^2 **(B)** 9 m/s^2 **(C)** 10 m/s^2 **(D)** 13.5 m/s^2

13. Identify the logic operation carried out.



(A) NOR **(B)** NAND
(C) OR **(D)** AND

14. A satellite can be in a geostationary orbit around earth at a distance r from the centre. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the centre is:

(A) $\frac{r}{2}$ **(B)** $\frac{r}{2\sqrt{2}}$ **(C)** $\frac{r}{(4)^{1/3}}$ **(D)** $\frac{r}{(2)^{1/3}}$

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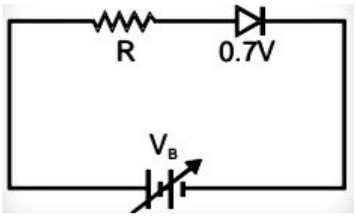
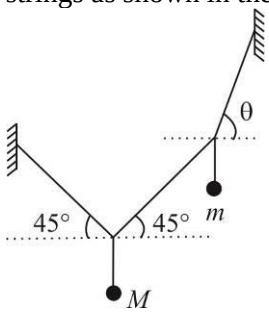
15. Assume that the junction diode in the following circuit requires a minimum current of 1 mA to be above the knee point (0.7V) of its I-V characteristic curve. Also assume that the voltage across the diode is independent of current above the knee point. If V_B = 5V, what should be the maximum value of R so that the voltage is above the knee point:-



(A) 4.3 kΩ **(B)** 860 kΩ **(C)** 4.3 Ω **(D)** 860 Ω

16. Two masses m and M are attached to strings as shown in the figure. In equilibrium tanθ is:



(A) $1+\left(\frac{2M}{m}\right)$ **(B)** $1+\left(\frac{2m}{M}\right)$ **(C)** $1+\left(\frac{M}{2m}\right)$ **(D)** $1+\left(\frac{m}{M}\right)$

17. A cylindrical capacitor has charge Q and length L. If both the charge and length of the capacitor are doubled, by keeping other parameters fixed, the energy stored in the capacitor:

(A) Remains same **(B)** Increases two times
(C) Decreases two times **(D)** Increases four times

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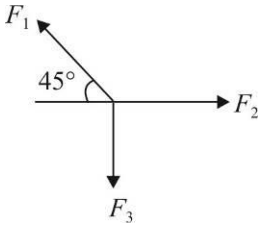
18. Three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are represented as shown. Each of them is equal in magnitude.



Column-I (Combination)		Column-II (Approximate Directions)	
(a)	$\vec{F}_1 + \vec{F}_2 + \vec{F}_3$	**(p)**	
(b)	$\vec{F}_1 - \vec{F}_2 + \vec{F}_3$	**(q)**	
(c)	$\vec{F}_1 - \vec{F}_2 - \vec{F}_3$	**(r)**	
(d)	$\vec{F}_2 - \vec{F}_1 - \vec{F}_3$	**(s)**	

(A) (a – r); (b – q); (c –s); (d – p) **(B)** (a – q); (b – r); (c – p); (d – s)

(C) (a – s); (b – p); (c – r); (d – q) **(D)** (a – p); (b – q); (c – s); (d – r)

19. A cube (density 0.5 *gm/cc*) of side 10 *cm* is floating in water kept in a cylindrical beaker of base area 1500 cm^2. When a mass m is kept on wooden block the level of water rises in the beaker by 2*mm*. Find the mass *m* (in *gm*).

(A) 100 **(B)** 200 **(C)** 300 **(D)** 400

20. An equilateral triangular wire frame is made from 3 rods of equal mass and length *l* each. The frame is rotated about an axis perpendicular to the plane of the frame and passing through its end. What is the radius of gyration of the frame?

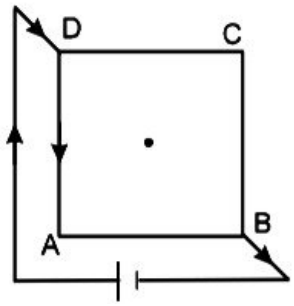
(A) $\frac{l}{2}$ **(B)** l **(C)** $\frac{l}{\sqrt{2}}$ **(D)** $\frac{l}{2\sqrt{3}}$

SPACE FOR ROUGH WORK

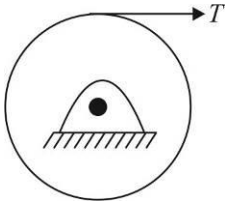
SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted.
The answer to each question should be **rounded off to the nearest integer**.

1. Figure shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points B and D as shown in the figure



2. A wheel 4*m* in diameter rotates about a fixed frictionless horizontal axis, about which its moment of inertia is 10 $kg\ m^2$. A constant tension of 40N is maintained on a rope wrapped around the rim of the wheel. If the wheel starts from rest at t = 0s, find the length of rope (in *m*) unwounded till t = 3*s*.



3. A very small sphere of mass 80 *gm* having a charge *q* is held at height 9*m* vertically above the centre of a fixed sphere of radius 1*m*, carrying an equal charge *q*. When released it falls until it comes to rest just before it comes in contact with the sphere. Calculate the charge *q*. (in μC) (g = 9.8 m/s^2) $\left(\frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\text{Nm}^2/\text{C}^2\right)$

4. A stone is tied to a light string whose other end is attached to the ceiling. The stone is displaced by an angle $\theta(<90°)$ and released. Find the value of θ in degree (to the closest integer) such that acceleration of the stone at the point of release is equal to the acceleration of the stone at the bottom most point in its path.

5. Two electric bulbs marked 25W – 200V and 100W – 200V are connected in series across a 200 V power supply. Find the combined power supply of the two bulbs in watt.

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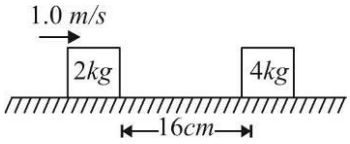
6. A disc of radius $2m$ and mass $5kg$ is pivoted at the rim and is set for small oscillations in vertical plane. If simple pendulum has to have the same period as that of the disc, the length of the simple pendulum (in metre) should be _____ .

7. Two L-C circuits have capacitances in the ratio 1:4. The two capacitors are charged to the same potential and then made to oscillate. If the maximum current in two circuits are in the ratio of 1:6 respectively, in what ratio should be the inductances of the two circuits?

8. We measure the period of oscillation of a simple pendulum. In successive measurements, the readings turn out to be 2.63 s, 2.56s, 2.42s, 2.71s and 2.78 s. Calculate the percentage error rounded off to the nearest integer.

9. In a uniform magnetic field of induction $2T,$ a wire in the form of semicircle of radius $1m$ rotates about the diameter of the circle with angular frequency $6rad/s.$ The total resistance of the circuit is $5ohm.$ Find the mean power generated (in watts) per period of rotation. (Take $\pi^2 = 10$)

10. The friction coefficient between the horizontal surface and each of the block shown in the figure is 0.2. The collision between the blocks is perfectly elastic. Find the separation (in *cm*) between them when they come to rest. ($g = 10 m/s^2$)



SPACE FOR ROUGH WORK

PART II : CHEMISTRY 100 MARKS

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE CHOICE is correct.

1. In Gabriel phthalimide synthesis, amine is always

(A) Alphatic primary amine **(B)** Alphatic secondary amine

(C) Aromatic primary amine **(D)** Aromatic secondary amine

2. Among the given oxides:

(i) CrO_5 (ii) V_2O_5 (iii) Al_2O_3 (iv) Na_2O

The correct increasing order of oxidation state of metal is:

(A) $(i) < (ii) < (iii) < (iv)$ **(B)** $(ii) < (iii) < (i) < (iv)$

(C) $(iii) < (iv) < (i) < (ii)$ **(D)** $(iv) < (iii) < (ii) < (i)$

3. Arrange the following ions in the decreasing order of size $F^-, O^{2-}, Mg^{2+}, A1^{3+}$.

(A) $O^{2-} > F^- > Mg^{2+} > Al^{3+}$ **(B)** $F^- > O^{2-} > Al^{3+} > Mg^{2+}$

(C) $F^- > Mg^{2+} > Al^{3+} > O^{2-}$ **(D)** $O^{2-} > Al^{3+} > F^- > Mg^{2+}$

4. Which of the following does not exist?

(A) SiF_6^{2-} **(B)** $SiBr_6^{2-}$ **(C)** $GeCl_6^{2-}$ **(D)** $Sn(OH)_6^{2-}$

5. Which of the following complex is active in magnetic field?

(A) $\left[Fe(CN)_6\right]^{3-}$ **(B)** $\left[Co(CN)_6\right]^{3-}$

(C) $\left[Fe(CN)_6\right]^{4-}$ **(D)** $\left[Fe(CO)_5\right]$

6. Which resonating structure of vinyl chloride is least stable?

(A) $CH_2 = CH - Cl$ **(B)** $\overset{-}{C}H_2 - CH = \overset{+}{C}l$

(C) $\overset{-}{C}H_2 - \overset{+}{C}H - Cl$ **(D)** All have equal stability

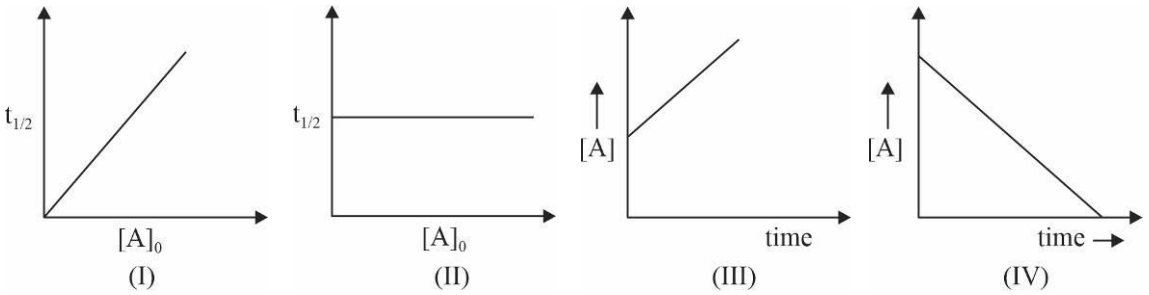
SPACE FOR ROUGH WORK

7. In the Hoffmann-Bromamide reaction intermediate species involved is/are

(A) R – CONHBr **(B)** $[R-\overset{\overset{O}{\|}}{C}-\overline{N}-Br]Na^-$

(C) R – N = C = O **(D)** All of these

8. From the graphs given below. Select the correct statements for zero order reaction $A \rightarrow B$.



(A) II, IV **(B)** II, III **(C)** III, IV **(D)** I, IV

9. Which of the following protein is soluble in water?

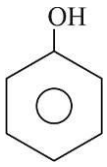
(A) Keratins **(B)** Elastins **(C)** Myosin **(D)** Insulin

10. Which of the following is not used for drying agent?

(A) $Br_2C=CBr_2$ **(B)** Silica gel

(C) Anhydrous $CaCl_2$ **(D)** Conc. H_2SO_4

11. C_2H_5MgBr react with which of the following to give Ethane gas?

(A) ROH **(B)** 

(C) NH_3 **(D)** All of these

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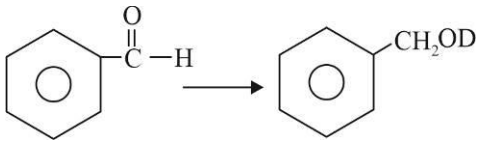
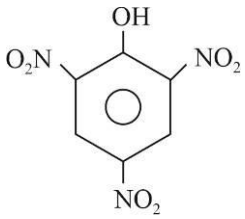
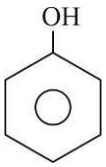
12. 

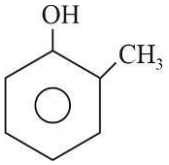
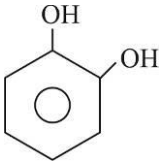
(i) $LiAlD_4, D_2O$ (ii) $NaBD_4, H_2O$

(iii) $LiAlH_4, D_2O$ (iv) $NaBH_4, C_2H_5OD$

(A) (i) & (ii) **(B)** (ii) & (iii) **(C)** (iii) & (iv) **(D)** (i) & (iv)

13. Which of the following will react with $NaHCO_3$ and evolve CO_2 gas.

(A)  **(B)** 

(C)  **(D)** 

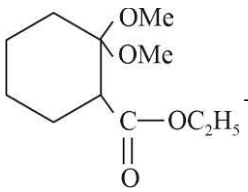
14. Amylose part of starch is water soluble. This is because:

(A) It is a linear polymer of $\alpha - D$ glucose

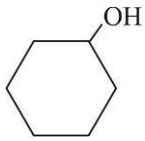
(B) It is a cross-linked polymer of $\alpha - D$ glucose

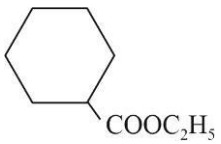
(C) It is a three dimensional network polymer of $\alpha - D$ glucose

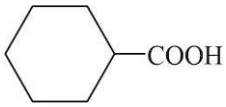
(D) It contains glycosidic linkage

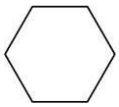
15.  $\xrightarrow[(ii)\ \Delta]{(i)\ \oplus H/H_2O} Y \xrightarrow[\Delta]{Zn-Hg/HCl} Z$

Finally obtained product is:

 **(A)**

 **(B)**

 **(C)**

 **(D)**

SPACE FOR ROUGH WORK

16. Given below are two statements, one is labeled as:
Assertion :(A) and other is labeled as Reason (R):
In the light of the above statement, choose the correct answer from the options given below
Assertion: Only primary amines give carbylamine reaction
Reason: Primary amines have two hydrogens attached to nitrogen atom which can be removed to form isocyanide

(A) Both (A) and (R) true but (R) is not correct explanation of (A)

(B) (A) is false but (R) is true

(C) Both (A) and (R) true and (R) is correct explanation of (A)

(D) (A) is true but (R) is false

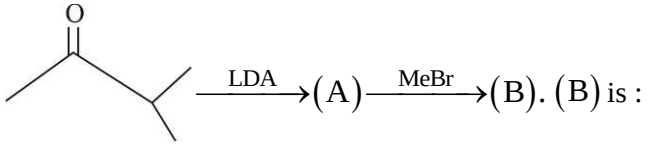
17. Statement-I: Compounds having only one chiral centre can have both enantiomer and diastereomer.
Statement-II: Diastereomer may or may not have chirality

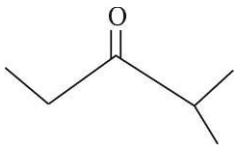
(A) Statement-I is True, Statement-II is True and Statement-II is a correct explanation for Statement-I

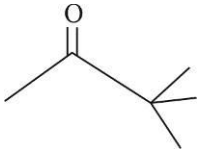
(B) Statement-I is True, Statement-II is True and Statement-II is NOT a correct explanation for Statement-I

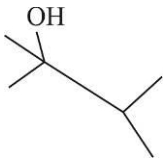
(C) Statement-I is True, Statement-II is False

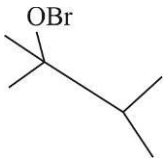
(D) Statement-I is False, Statement-II is True

18. 

(A) 

(B) 

(C) 

(D) 

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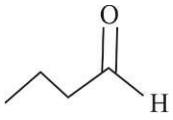
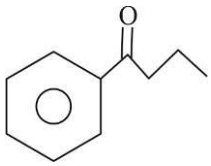
19. An organic compound $'A'\ C_5H_{10}$ on treatment with $K_2Cr_2O_7 / H^{\oplus}$ yield compound $'B'\ C_4H_8O$. Compound $'A'$ also yields compound $'B'$ on reductive ozonolysis. Compound $'A'$ is:

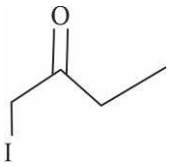
(A) $2-$methylbutene **(B)** $3-$methylbutene

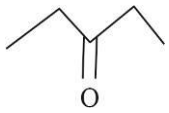
(C) $2-$methyl butane **(D)** $3-$methyl pentene

20. Which of the following compound will give Iodoform test?

(A) 

(B) 

(C) 

(D) 

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SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted.
The answer to each question should be **rounded off to the nearest integer**.

1. In the combustion of propane $54gm$ of H_2O is produced. The amount (in gm) of propane taken initially is $[X]\times10^{-1}$. The value of 'X' is:

2. $X(aq)+Y(aq)\rightleftharpoons 2Z(aq)$

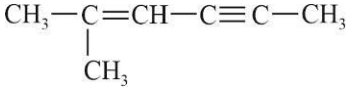
Initially 2 mole each of X, Y and Z are taken in 1 litre vessel. Equilibrium constant is 64. The concentration of Z at equilibrium is $[X]\times10^{-1}mol\ L^{-1}$. The value of 'X' is:

3. Find the molarity of $Sn^{2+}(10mL)$ required to reduce $30ml$ of $1.0M\ K_2Cr_2O_7$ solution:

4. Empirical formula of a given octahedral complex is $CrBr_3\cdot3NH_3\cdot2H_2O$. It precipitates 2 moles of $AgBr$ with $AgNO_3$, then number of Br atoms which satisfies secondary valency of central atom is:

5. For a process $\Delta H_{sub}=101\ kcal\ mol^{-1}$ and $\Delta H_{vap}=99\ kcal\,mol^{-1}$. Then ΔH_{fusion} (in $kcal\ mol^{-1}$) will be:

6. Determine number of σ bonds in the given compound.



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7. A source of monochromatic radiation of wavelength 400nm provides 400J of energy in 4 seconds. When this radiation falls on the surface of sodium, $x\times10^{20}$ electrons are ejected per second. Assume that wavelength is 400nm and it is sufficient for ejection of electron from the surface of sodium metal. The value of x is …… (Nearest integer) $\left(h=6.626\times10^{-34}\text{Js}\right)$

8. A factory owner used $2.00\times10^{3}\,\text{m}^3$ ethane $\left(C_2H_6\right)$ gas, (assume C_2H_6 is an ideal gas) in a year to heat his home. Under the pressure of 2.0 atm and 350K mass of gas used is $x\times10^{5}\text{g}$. The value of x is …. (Nearest integer) (Given $R=0.083\text{L atm K}^{-1}\text{ mol}^{-1}$)

9. A gas Q is bubbled through water during a soft drink manufacturing process at 298K. If Q exerts a partial pressure of 0.835 bar then x mmol Q would dissolve in 0.9L of water. The value of x is ……. (Nearest integer)

(Henry's law constant for Q at 298K is 1.67×10^{3} bar, $d_{H_2O}=1\text{g mL}^{-1}$)

10. Consider the cell at $25°\text{C}$.

$$X,X^{2+}(aq),(1M)\,\|\,Y^{3+}(aq),Y^{2+}(aq)\,|\,Pt(s)$$

The ratio of $\frac{Y^{2+}}{Y^{3+}}$ at the cell potential of 1.5 V is x. The value of x is …….. (Nearest integer)

(Given $E^0_{Y^{3+}/Y^{2+}}=0.77\text{V}, E^0_{X^{2+}/X}=-0.76\text{V}$, $\frac{2.303\text{ RT}}{F}=0.0591$)

SPACE FOR ROUGH WORK

PART III : MATHEMATICS 100 MARKS

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE CHOICE is correct.

1. The area (in sq. units) of the region, given by the set $\left\{(x, y) \in R \times R \mid x \leq 0, 3x^2 + 2 \leq y \leq 8x + 5\right\}$ is:

(A) $\frac{14}{25}$ **(B)** $\frac{25}{14}$ **(C)** $\frac{14}{27}$ **(D)** $\frac{27}{14}$

2. The locus of the centroid of the triangle formed by any point P on the hyperbola $25x^2 - 16y^2 + 150x - 64y - 239 = 0$ and its foci is:

(A) $225x^2 - 144y^2 + 1350x + 576y - 1049 = 0$

(B) $225x^2 - 144y^2 - 1350x - 576y + 10494 = 0$

(C) $225x^2 - 144y^2 + 1350x + 576y + 1049 = 0$

(D) $225x^2 - 144y^2 + 1350x - 576y + 1049 = 0$

3. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{5n} = 4S_{2n}$, then the value of $\frac{S_{3n}}{S_{2n}}$ is:

(A) 2 **(B)** $9/5$ **(C)** $5/9$ **(D)** $3/2$

4. Let 11 distinct balls distributed among 5 boxes, B_1, B_2, B_3, B_4 and B_5. If the probability that B_2 contains exactly 2 balls is $k\left(\frac{4}{5}\right)^{11}$ then k lies in the set:

(A) $\{x \in R : |x-1| < 1\}$ **(B)** $\{x \in R : |x-2| < 1\}$

(C) $\{x \in R : |x-3| < 1\}$ **(D)** $\{x \in R : |x-5| < 1\}$

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5. A line is drawn form A(–2, 0) to intersect the curve $y^2 = 4x$ at P and Q in the first quadrant such that $\frac{1}{AP}+\frac{1}{AQ}<\frac{1}{4}$. Then the slope of the line is always

(A) > 3 **(B)** $<1/\sqrt{3}$ **(C)** $>\sqrt{2}$ **(D)** $>1/\sqrt{3}$

6. The values of a and b, for which the system of equations

$$3x-4y+8=5$$
$$2x+ay-1=2$$
$$3x+y-2=b$$

has no solution, are:

(A) $a=\frac{1}{2}, b\neq\frac{55}{19}$ **(B)** $a=\frac{1}{2}, b=\frac{55}{19}$

(C) $a\neq\frac{1}{2}, b=\frac{55}{19}$ **(D)** $a\neq\frac{1}{2}, b\neq\frac{55}{19}$

7. Let $f: R\to R$ be defined as

$$\begin{Bmatrix} \frac{\delta\left|5x-4-x^2\right|}{\left(4-5x+x^2\right)} & , & x>1 \\ \frac{\lambda\sin(1-x)}{[x-1](x-1)}+2 & , & x<1 \\ 3\lambda & , & x=1 \end{Bmatrix}$$

where $[x]$ is the greatest integer less than or equal to x. If f is continuous at $x=1$, then $\lambda+\delta$ is equal to:

(A) -2 **(B)** 2 **(C)** 1 **(D)** 3

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8. The expression $\log_2 5-\sum_{k=1}^{4}\log_2\left(\sin\left(\frac{k\pi}{5}\right)\right)$ reduces to $\frac{p}{q}$, where p and q are co-prime, the value of p^2-q^2 is

(A) 13 **(B)** 17 **(C)** 26 **(D)** 29

9. Let complex number α and $\frac{1}{\alpha}$ lie on circles $(x-x_0)^2+(y-y_0)^2=r^2$ and $(x-x_0)^2(y-y_0)^2=4r^2$, respectively. If $z_0=x_0+iy_0$ satisfies the equation $2|z_0|^2=r^2+2$, then $|\alpha|$ =

(A) $\frac{1}{\sqrt{2}}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{\sqrt{7}}$ **(D)** $\frac{1}{3}$

10. The value of the definite integral $\int_{\pi/36}^{5\pi/36}\frac{(\sec 3x)^{1/5}\,dx}{(\operatorname{cosec}3x)^{1/5}+(\sec 3x)^{1/5}}$ is:

(A) $\frac{\pi}{12}$ **(B)** $\frac{\pi}{9}$ **(C)** $\frac{\pi}{18}$ **(D)** $\frac{\pi}{2}$

11. Let the vectors $(a+b+2c)\hat{i}+a\hat{j}+b\hat{k}, c\hat{i}+(b+c+2a)\hat{j}+b\hat{k}$ and $c\hat{i}+a\hat{j}+(c+a+2b)\hat{k}$ are coplanar, then which of the following is true?

(A) $a+b+c=1$ **(B)** $a+b+c=0$

(C) $a=b+c$ **(D)** $c=a+b$

12. Let $g:N\to N$ defined as

$g(\alpha)=\beta,$

$g(\beta)=\gamma,$

$g(\gamma)=\alpha$ where $\alpha,\beta,\gamma\in N$

Then which of the following statements is true?

(A) $gogog=g$

(B) There exists a function $f:N\to N$ such that $gof=f$

(C) There exists a one-one function $f:N\to N$ such that $fog=f$

(D) None of these

SPACE FOR ROUGH WORK

13. Let ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a^2>b^2$ passes through $\left(2,\frac{2\sqrt{5}}{3}\right)$ and has eccentricity $\sqrt{5}/3$. If a circle centered at focus $F(\alpha,0), \alpha>0$ of E and $(\text{radius})^2=\frac{101+36\sqrt{5}}{9}$, intersects E at two points P and Q, then PQ^2 is equal to:

(A) $\frac{80}{9}$ **(B)** $\frac{16}{3}$ **(C)** $\frac{82}{3}$ **(D)** $\frac{14}{3}$

14. Let $f(x)=\begin{cases}\text{sgn}(x)+x, & -\infty<x<0\\ -1+\sin x, & 0\le x<\pi/2,\\ \cos x, & \pi/2\le x<\infty\end{cases}$ then the number of points, where $f(x)$ is not differentiable, is/are:

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

15. Let $f(x)=\frac{3}{2}\sin^4 x+\left(4+\sqrt{3}\right)\sin^3 x+3\sqrt{3}\sin^2 x-2$, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. then, f is:

(A) Increasing in $\left(\frac{-\pi}{3},0\right)$ **(B)** Decreasing in $\left(\frac{-\pi}{2},\frac{\pi}{3}\right)$

(C) Increasing in $\left(\frac{-\pi}{6},0\right)$ **(D)** Decreasing in $\left(\frac{-\pi}{3},0\right)$

16. The number of real roots of the equation $e^{8x}-2e^{6x}+e^{4x}-4e^{2x}+4=0$ is:

(A) 4 **(B)** 3 **(C)** 2 **(D)** 0

17. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}=3-2x\,e^{6x-2y}, x\in\left(\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right), y(0)=0$ then $2y\left(\frac{1}{2}\right)+\ln(2)$ is equal to:

(A) 2 **(B)** 0 **(C)** 3 **(D)** 1

SPACE FOR ROUGH WORK

18. If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in identity $\frac{1}{a+b}+\frac{1}{a+2b}+\frac{1}{a+3b}+....+\frac{1}{a+nb}=\alpha n+\beta n^2+\gamma n^3$ Then the value of $\alpha-\beta$ is:

(A) $\frac{3a^3-2b^2}{3a^2}$ **(B)** $\frac{3a^3-b^2}{3a^3}$ **(C)** $\frac{3a^2-b^3}{3a^3}$ **(D)** $\frac{3a^2-b^2}{3a^3}$

19. A train timetable must be compiled for various day of the week so that two trains twice a day depart for three days, one train daily for two days, and three trains once a day for two days. How many different timetables can be complied?

(A) 140 **(B)** 210 **(C)** 133 **(D)** 72

20. If a_1 and a_2 are two values of a for which the unit vector $a\hat{i}+b\hat{j}+\frac{1}{2}\hat{k}$ is linearly dependent with $\hat{i}+2\hat{j}$ and $\hat{j}-2\hat{k}$, then $\frac{1}{a_1}+\frac{1}{a_2}$ is equal to

(A) 1 **(B)** $\frac{1}{8}$ **(C)** $\frac{-16}{11}$ **(D)** $\frac{-11}{16}$

SPACE FOR ROUGH WORK

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted.
The answer to each question should be **rounded off to the nearest integer**.

1. If the value $\left(\frac{1}{2}+\frac{4}{2^2}+\frac{7}{2^3}+\frac{10}{2^4}+.....\infty\right)^{\log_{\frac{2}{3}}\left(\frac{2}{5}+\frac{2^2}{5^2}+\frac{2^3}{5^3}+.....\right)}$ is l, then l is equal to _____ .

2. There are 6 students in class 10, 7 students in class 11 and 8 students in class 12. If the number of ways, in which 12 students can be selected from them so as to include at least 3 students from each class and at most 7 students from the total 13 students of class 10 and 11 is $10000k$, then $[k]$ equal to _____

{[.] denotes greatest integer function}

3. If α,β are roots of the equation $3x^2-\left(19\sqrt{5}\right)x+7^{1/4}=0$, $\alpha>\beta$ and $P_n=\alpha^n-\beta^n$ and for each positive integer n, then the value of $\left(\frac{P_{13}P_{18}-19\sqrt{5}P_{13}P_{16}}{P_{15}P_{16}-19\sqrt{5}P_{13}P_{16}}\right)+\left(\frac{P_2}{\alpha-\beta}\right)\left(\frac{3}{19\sqrt{5}}\right)$ is equal to

4. If the primitive the function $f(x)=\frac{x^{2009}}{\left(1+x^2\right)^{1006}}$ w.r.t. x is equal to $\frac{1}{n}\left(\frac{x^2}{1+x^2}\right)^m+C$, then $\frac{n}{m}$ is equal to _____

SPACE FOR ROUGH WORK

5. Let $\vec{p}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{q}=2\hat{i}+3\hat{j}-\hat{k}$ be two vectors. If a vector $\vec{r}=\left(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}\right)$ is perpendicular to each of the vectors $\left(\vec{p}\right)$ and $\left(2\vec{p}-3\vec{q}\right)$ and $\left|\vec{r}\right|=3\sqrt{3}$, then $|\alpha|+|\beta|+|\gamma|$ is equal to _____

6. Let numbers $S=\left\{n\in W\left|\begin{pmatrix}0 & 1\\ i & 0\end{pmatrix}^n\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}\forall a,b,c,d\in R\right.\right\}$, where $i=\sqrt{-1}$ and W is a set of whole numbers. If λ_1 and λ_2 are the number of $3-$ digit numbers and $1-$ digit numbers respectively such that $\lambda_1+\lambda_2=19\lambda,$ then the value of λ will be

7. The ratio of the sum of the coefficients of the middle terms in the expansion of $(1+x)^{15}$ and the coefficient of middle term in expansion of $(1+x)^{16}$ is _____ .

8. The sum of the digits of the term independent of $'x'$ in the expansion of

$$\left[9^{\log_3\sqrt{3+x^{1/3}}}-4^{\frac{3}{2}\log_4\left(\frac{3\sqrt{x}+1}{\sqrt{x}}\right)^{2/3}}\right]^{10}$$

, where $x\neq 0,$ is equal to:

9. Let $M=\left\{A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}:a,b,c,d\in\{\pm3,\pm2,\pm1,0\}\right\}$. Define $f:M\to Z,$ as $f(A)=\det(A),$ for all $A\in M,$ where Z is set of all integers. Then the number of $A\in M$ such that $f(A)=10$ is equal to $8\lambda.$ The value of λ is:

10. Consider the following frequency distribution:

Class:	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$
Frequency	50	p	20	114	q

If the sum of all frequencies is 534 and median is 32, then $\frac{p-q}{19}$ is equal to _____ .

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••• End of JEE Main Home Practice Test - 6 [JEE - 2024] •••